

Linear Equation Solve by Matrix Method

Ex. Find the solution of the system of equation

$$2x + 6y + 11 = 0$$

$$6x + 20y - 6z + 3 = 0$$

$$6y - 18z + 1 = 0$$

Soln. Given equations also written as

$$2x + 6y + 0z = -11$$

$$6x + 20y - 6z = -3$$

$$0x + 6y - 18z = -1$$

$$\therefore A = \begin{bmatrix} 2 & 6 & 0 \\ 6 & 20 & -6 \\ 0 & 6 & -18 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -11 \\ -3 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 & 0 \\ 6 & 20 & -6 \\ 0 & 6 & -18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -11 \\ -3 \\ -1 \end{bmatrix}$$

By $R_2 \rightarrow R_2 - 3R_1$

$$\Rightarrow \begin{bmatrix} 2 & 6 & 0 \\ 0 & 2 & -6 \\ 0 & 6 & -18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -11 \\ +30 \\ -1 \end{bmatrix}$$

By $R_3 \rightarrow R_3 - 3R_2$

$$\Rightarrow \begin{bmatrix} 2 & 6 & 0 \\ 0 & 2 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -11 \\ 30 \\ -91 \end{bmatrix}$$

$$\Rightarrow 2x + 6y = -11$$

$$2y - 6z = 30$$

$$0x + 0y + 0z = -91 \rightarrow \text{is meaningless.}$$

②

⇒ The given system of equations is inconsistent.

Q. Solve $x + y + z = 6$, $2x + y - 3z = -5$, $3x - 2y + z = 2$.

Soln - Here, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ 3 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ -5 \\ 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ 3 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -1 & -4 \\ 3 & -5 & 3 \end{vmatrix} = -23$$

Now, we find cofactors

$$A_1 = \begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix} = -5, \quad A_2 = -3, \quad A_3 = -4,$$

$$B_1 = -11, \quad B_2 = -2, \quad B_3 = 5, \quad \text{and } C_1 = -7, \quad C_2 = 5, \quad C_3 = -1$$

$$\therefore A^{-1} = \frac{1}{|A|} \begin{bmatrix} -5 & -3 & -4 \\ -11 & -2 & 5 \\ -7 & 5 & -1 \end{bmatrix} = \frac{1}{23} \begin{bmatrix} 5 & 3 & 4 \\ 11 & 2 & -5 \\ 7 & -5 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{23} \begin{bmatrix} 5 & 3 & 4 \\ 11 & 2 & -5 \\ 7 & -5 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ -5 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{23} \begin{bmatrix} 5 \times 6 + 3 \times (-5) + 4 \times 2 \\ 11 \times 6 + 2 \times (-5) + (-5) \times 2 \\ 7 \times 6 + (-5) \times (-5) + 1 \times 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{23} \begin{bmatrix} 23 \\ 46 \\ 69 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow x = 1, \quad y = 2, \quad z = 3$$

Solved.